NPS Learning in Place

Algebra II

Name______________School__________________Teacher _________________

Week 1	**Statistics**
Week 2	**Statistics** **Review**

This is taken from the VDOE Formula page that you are able to use when taking assessments. You do not have to memorize these formulas.

Statistics Formulas:

Given:

x represents an element of the data set,

x_i represents the i^{th} element of the data set,

n represents the number of elements in the data set,

μ represents the mean of the data set,

σ represents the standard deviation of the data set, and

σ^2 represents the variance of the data set

z-score: $z = \dfrac{x - \mu}{\sigma}$

standard deviation: $\sigma = \sqrt{\dfrac{\sum_{i=1}^{n}(x_i - \mu)^2}{n}}$

variance $(\sigma^2) = \dfrac{\sum_{i=1}^{n}(x_i - \mu)^2}{n}$

Day 1

KeyConcept Measures of Central Tendency

Use	Which Is…	When…
mean	the sum of the data divided by the number of items in the data set	The data set has no outliers.
median	the middle number of the ordered data, or the mean of the middle two numbers	The data set has outliers, but there are no big gaps in the middle of the data.
mode	the number or numbers that occur most often	The data set has many repeated numbers.

Which measure of central tendency best represents the data at the right? Explain.

Since there are outliers and no big gaps in the middle, the median best represents the data.

16	17	15	17
12	16	16	15
2	18	18	18
40	16	48	1

Find the Mode: __________

WEATHER The table below shows daytime high temperatures for a week.

Day	Sun.	Mon.	Tues.	Wed.	Thurs.	Fri.	Sat.
Temperature	64°F	73°F	69°F	70°F	71°F	75°F	74°F

Find the mean and median for this data. Mean: ______________ Median: __________________

Example 2 Samples Versus Populations

Determine whether each of the following represents a *population* or a *sample*.

a. The Nielsen Poll estimates the average number of hours of television watched per week for U.S. households.

This represents a sample because only a fraction of U.S. residents are polled.

b. A mathematics exam is given to every graduating senior in the country to analyze certain mathematics skills.

This represents a population because the exam tests *every* graduating senior.

Guided Practice

2A. A teacher compares the scores on a test in her class.

2B. A teacher compares her class with the rest of the country on a national test.

Determine whether each of the following represents a *population* or a *sample*.

13. Carissa calculates the average number of pineapples in 25 cans of pineapple.

14. The IRS calculates the mean income per household.

15. Middleburg Elementary School calculates the average height of all of its students.

16. Members of the football team want to compare their times in the 40 meter dash to those of the rest of the conference.

17. Jermaine asks 100 random people at the mall for their opinions on education.

18. The NFL compares the yards per game allowed by each team's defense.

19. Tomás compares the populations of every state.

20. Dona asks 400 random people what their favorite season is.

Day 2

> **StudyTip**
> **Population Mean**
> When the population mean μ is known, then the symbol can be interchanged with $\bar{x}$.

2 Measures of Variation

Measures of variation describe the *dispersion* or spread of a set of data. Two common measures of variation are the **variance** and **standard deviation**. These measures describe how closely a set of data clusters about the mean.

The sample mean $\bar{x}$, read *x bar*, and the population mean μ, or mu, are calculated the same way. The formulas for calculating the sample standard deviation s and the population standard deviation σ, or sigma, are given below.

KeyConcept Standard Deviation Formulas

Sample	Population
$s = \sqrt{\dfrac{\sum_{k=1}^{n}(x_k - \bar{x})^2}{n-1}}$	$\sigma = \sqrt{\dfrac{\sum_{k=1}^{n}(x_k - \mu)^2}{n}}$

Real-World Example 4 Standard Deviation

TEST SCORES The Chapter 3 and Chapter 4 scores from Mr. Hoff's class both have a mean of 75. Find and compare their standard deviations.

Mr. Hoff's 2nd Period Chapter 3 Scores	Mr. Hoff's 2nd Period Chapter 4 Scores
85, 80, 75, 75, 70, 75, 75, 65, 75, 75, 75, 80, 75, 75, 70, 80, 70, 75, 75, 75, 75, 75, 75	100, 100, 90, 10, 100, 95, 10, 95, 100, 100, 85, 15, 95, 20, 95, 90, 100, 100, 90, 10, 100, 100, 25

a. Find the standard deviation for the Chapter 3 scores.

Step 1 This is a population. Since the mean of each set was 75, $\mu = 75$.

Step 2 Find the standard deviation.

$$\sigma = \sqrt{\frac{\sum_{k=1}^{n}(x_k - \mu)^2}{n}} \quad \text{Standard Deviation Formula}$$

$$= \sqrt{\frac{(85-75)^2 + (80-75)^2 + \ldots + (75-75)^2 + (75-75)^2}{23}} \approx 3.9$$

The class mean of the Chapter 3 test is 75 with a standard deviation of about 3.9.

You Try It!

Calculate the mean and standard deviation of the population of data.

Change 30 to 70. What should happen to the mean and standard deviation? Recalculate to confirm your results.

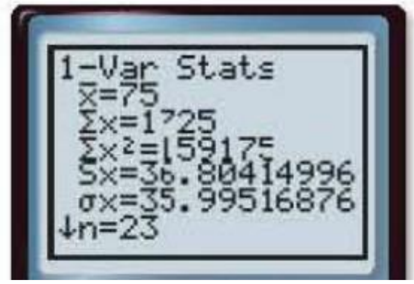
28	34	33	33	31
33	29	34	36	31
30	29	32	28	36
29	33	29	28	28
26	31	28	27	29

Use a calculator to find the standard deviation of the Chapter 4 scores.

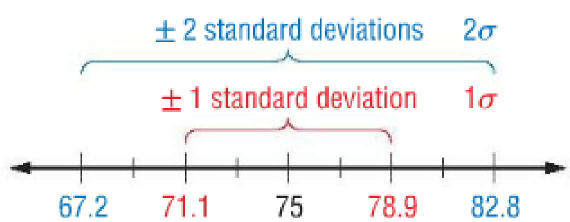
Clear all lists. Then press STAT ENTER and enter each data value, pressing ENTER after each value.

To view the statistics, press STAT ▶ 1 ENTER.

The class mean of the Chapter 4 test is 75 with a standard deviation of about 36.



In a given set of data, the majority of the values fall within one standard deviation of the mean. Almost all of the data will fall within 2 standard deviations. Mr. Hoff's Chapter 3 scores had a mean of 75 and a standard deviation σ of 3.9. We can illustrate this graphically on a number line.



StudyTip

Standard Deviation The greater the standard deviation, the more the data deviate from the mean.

If Mr. Hoff were to compare his students' scores with other students throughout the country on a national test, the class would be considered a sample of all of the students who took the test. He would then need to calculate a sample mean $\overline{x}$ and a sample standard deviation s.

EDUCATION Below are ACT scores for a recent year.

Mean ACT Scores by State									
20.2	21.3	21.5	20.4	21.6	20.3	22.5	21.5	17.8	20.5
20.0	21.7	21.3	20.2	21.6	22.0	21.6	20.3	19.8	22.6
20.8	22.4	21.4	22.2	18.8	21.5	21.7	21.7	21.2	22.5
21.2	20.1	22.3	20.3	21.2	21.4	20.6	22.5	21.8	21.9
19.3	21.5	20.5	20.3	21.5	22.7	20.9	22.5	22.2	21.4

Source: ACT, Inc.

a. Compare the mean and median of the data.

b. Is this a sample or a population?

c. Find the standard deviation of the data. Round to the nearest hundredth.

d. Suppose the state with a mean score of 20.0 incorrectly reported the results. The score for the state is actually 22.5. How are the mean and median of the data affected by this change?

STUDENT-TEACHER RATIOS The table at the right shows the number of students in every math class at Principal Johnson's high school.

a. Which measure of central tendency best represents the data? Why?

b. Is this a sample or a population?

c. Find the standard deviation of the data. Round to the nearest hundredth.

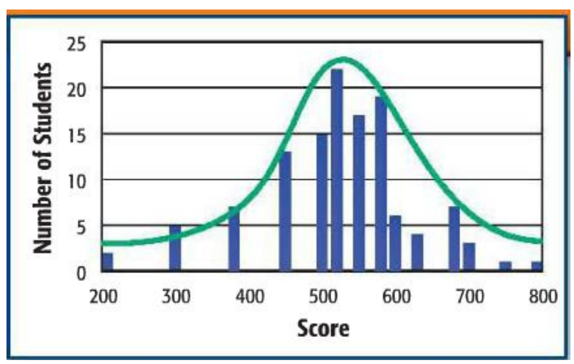
Students Per Math Class					
25	27	26	26	19	27
24	23	19	28	25	24
20	22	22	24	26	18
28	29	29	26	24	24
23	23	25	25	29	28

Journal Prompts: (answer on separate paper)

1) What are some ways you can measure the dispersion of a set of data?
2) What can the standard deviation could tell you about the data.

Day 3

The graph shows the Scholastic Assessment Test (SAT) math scores for Ms. Fuentes's students. The data are clustered in the center, and the graph is shaped like a bell. This discrete probability distribution is close to being *normally* distributed.

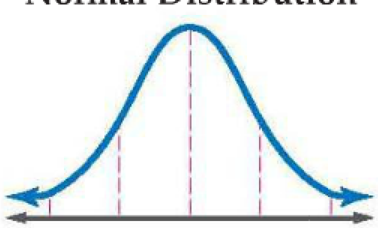


1 Normal and Skewed Distributions

In a **continuous probability distribution**, the

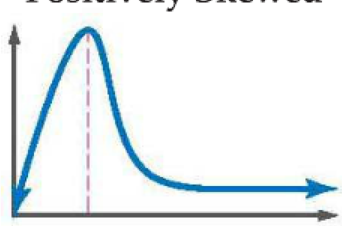
While the normal distribution is continuous, discrete distributions like the one above can have a *normal* shape. Distributions with other shapes are called **skewed distributions**.

Normal Distribution



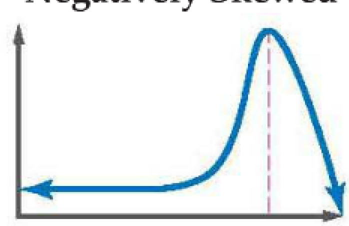
shaped like a bell and symmetric

Positively Skewed



mass of distribution at the left and tail to the right

Negatively Skewed



mass of distribution at the right and tail to the left

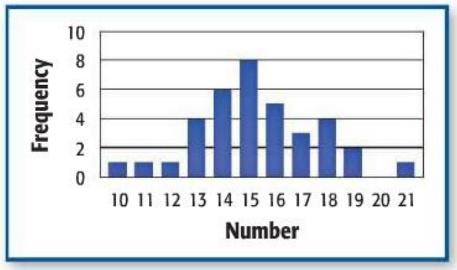
Example 1 Classify a Data Distribution

Determine whether the following data appear to be *positively skewed*, *negatively skewed*, or *normally distributed*.

a.

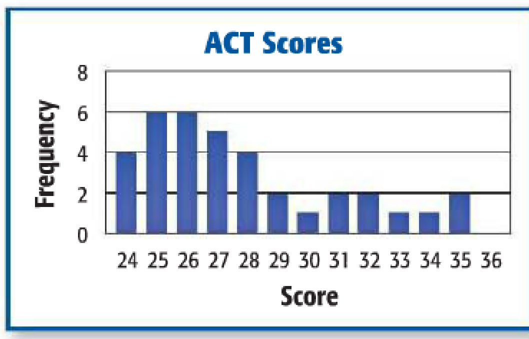
10	12	13	15	13	15	15	14	14	16	15	18	16	18	19	16	14	13
16	16	15	14	18	17	11	19	17	18	13	15	14	21	14	15	15	17

Use the frequency table to make a graph. Since the graph is high in the middle and appears to be somewhat symmetric, the data are normally distributed.



24	24	35	33	25	27	26	26	28	30	31	24	28	27	25	26	28	26
25	32	31	35	24	26	27	29	32	34	29	28	27	25	25	26	27	25

Use the frequency table to make a graph. Since the graph is high on the left and low in the middle and right, the data are positively skewed.



1. Determine whether the data at the right appear to be *positively skewed*, *negatively skewed*, or *normally distributed*.

Shoe Size	6	7	8	9	10	11	12
Frequency	4	8	9	7	4	2	3

Determine whether the data appear to be *positively skewed*, *negatively skewed*, or *normally distributed*.

4.

20 Most Visited National Parks	
Visitors (millions)	Number of Parks
3–4	10
4–5	2
5–6	2
6–7	1
7–8	1
8+	4

5.

Tallest Buildings in the World	
Stories	Number of Buildings
0–39	1
40–59	11
60–79	35
80–99	9
100+	6

Algebra II Learning in Place

Day 4

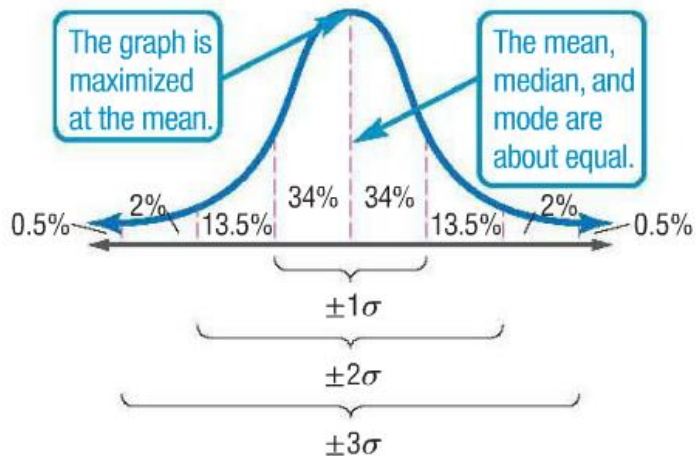
2 The Empirical Rule

The Empirical Rule describes other characteristics of normal distributions.

KeyConcept The Empirical Rule

A normal distribution with mean μ and standard deviation σ has the following properties.

- About 68% of the values are within 1σ of the mean.
- About 95% of the values are within 2σ of the mean.
- About 99% of the values are within 3σ of the mean.

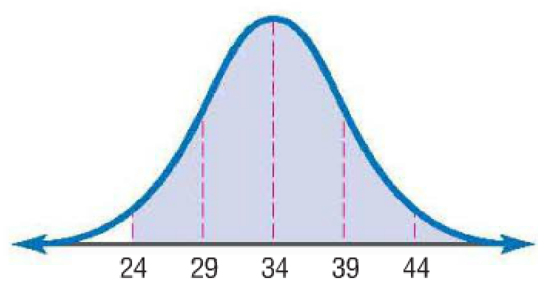


Example 2 Normal Distribution

A normal distribution of data has a mean of 34 and standard deviation of 5. Find the probability that random value x is greater than 24, that is, $P(x > 24)$.

$\mu = 34$ and $\sigma = 5$

The probability that a randomly selected value in the distribution is greater than $\mu - 2\sigma$, that is, $34 - 2(5)$ or 24, is the shaded area under the normal curve.



$$P(x > 24) = 13.5 + 34 + 34 + 13.5 + 2 + 0.5$$
$$= 97.5\%$$

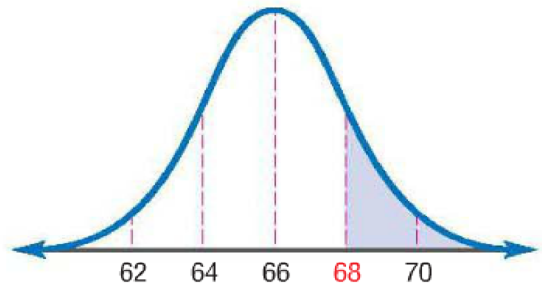
GuidedPractice

2. Find the probability that a randomly selected value in the distribution above is less than 49.

b. What is the probability that a teenager selected at random has a height greater than 68 inches?

From the curve, values greater than 68 are more than 1σ from the mean. 13.5% are between 1σ and 2σ, 2% are between 2σ and 3σ, and 0.5% are greater than 3σ.

So, the probability that a teenager selected at random has a height greater than 68 inches is $13.5 + 2 + 0.5$ or 16%.



Real-World Example 3 Normally Distributed Sample

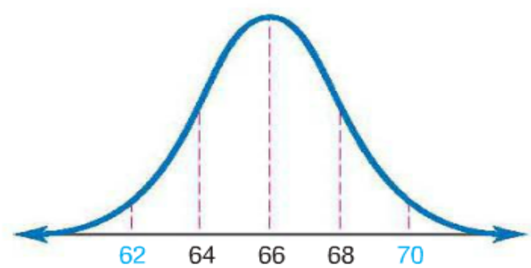
HEIGHTS **The heights of 1800 teenagers are normally distributed with a mean of 66 inches and a standard deviation of 2 inches.**

a. About how many teens are between 62 and 70 inches?

Draw a normal curve.

62 and 70 are 2σ away from the mean. Therefore, about 95% of the data are between 62 and 70.

Since $1800 \times 95\% = 1710$, we know that about 1710 of the teenagers are between 62 and 70 inches tall.



a. About what percent of the product lasts between 150 and 210 days?

Guided Practice

GRADES **The grade-point averages of 1200 students at East High School are normally distributed with a mean of 2.6 and a standard deviation of 0.6.**

3A. About how many students have a grade-point average between 2.0 and 3.2?

3B. What is the probability that a randomly selected student has an average less than 3.8?

Day 5

1. **ACT** The table at the right shows recent composite ACT scores. Determine whether the data appear to be *positively skewed*, *negatively skewed*, or *normally distributed*.

Score	% of Students
33–36	1
28–32	9
24–27	19
20–23	29
16–19	27
13–15	12

Source: ACT, Inc.

2. A normal distribution of data has a mean of 161 and standard deviation of 12. Find the probability that random value x is less than 149, that is $P(x < 149)$.

3. **SCHOOL** Mr. Bash gave a quiz in his social studies class. The scores were normally distributed with a mean of 21 and a standard deviation of 2.

   a. What percent would you expect to score between 19 and 23?

   b. What percent would you expect to score between 23 and 25?

   c. What is the probability that a student scored between 17 and 25?

Day 6

Normal Distribution Exercises

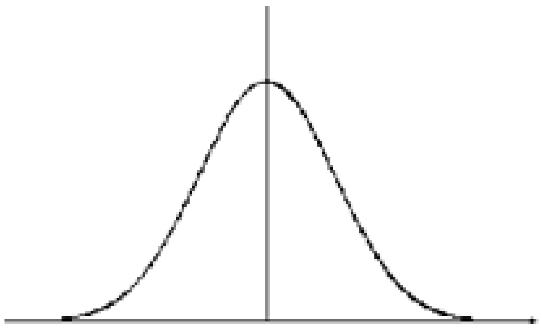
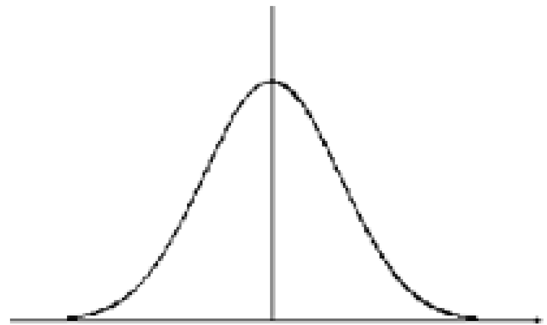
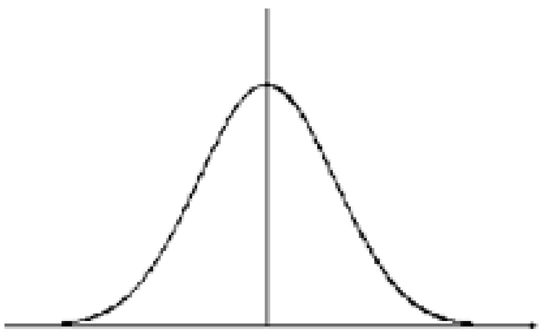
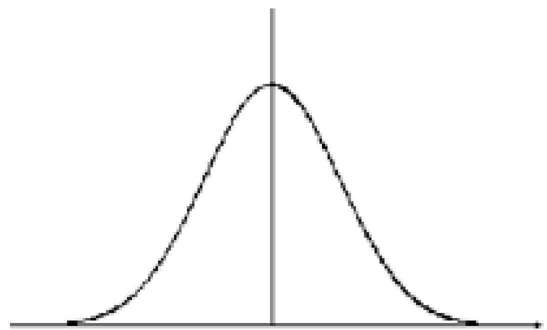
Represent each of the following distributions on one of the normal distribution graphs. For each, show three standard deviations to the left and three standard deviations to the right of the mean.

1. A normal distribution with a mean of 7 and a standard deviation of 2.

2. A normal distribution with a mean of 500 and a standard deviation of 100.

3. The weights of cattle at the fair this year were normally distributed, with a mean of 800 pounds and a standard deviation of 65 pounds.

4. The amount of time a middle school student studies per night is normally distributed with a mean of 30 minutes and a standard deviation of 7 minutes.

Day 7

The Empirical Rule is only useful for evaluating specific values, such as $\mu + \sigma$. Once the data set is *standardized*, however, any data value can be evaluated. Data are standardized by converting them to z-scores. The **z-score** represents the number of standard deviations that a given data value is from the mean. Therefore, z-scores can be used to determine the position of any data value within a set of data.

KeyConcept Formula for z-Scores

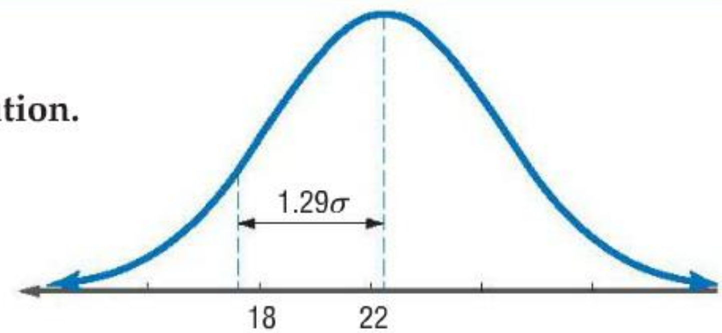
The z-score for a data value X in a set of normally distributed data is given by $z = \frac{X - \mu}{\sigma}$, where μ is the mean and σ is the standard deviation.

Example 1 Use z-Scores to Locate Position

Find z if $X = 18$, $\mu = 22$, and $\sigma = 3.1$.

Indicate the position of X in the distribution.

$z = \frac{X - \mu}{\sigma}$	Formula for z-scores
$= \frac{18 - 22}{3.1}$	$X = 18$, $\mu = 22$, $\sigma = 3.1$
≈ -1.29	Simplify.



The z-score that corresponds to $X = 18$ is approximately -1.29. Therefore, 18 is about 1.29 standard deviations less than the mean of the distribution.

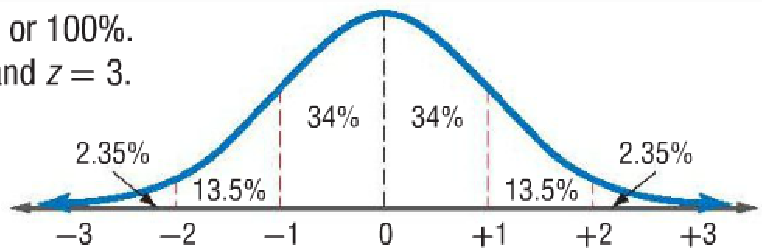
GuidedPractice

1. Find X if $\mu = 39$, $\sigma = 8.2$, and $z = 0.73$. Indicate the position of X in the distribution.

Any combination of mean and standard deviation is possible for a normally distributed set of data. As a result, there are infinitely many normal probability distributions. This makes comparing two individual distributions difficult. Different distributions *can* be compared, however, once they are standardized using z-scores. The **standard normal distribution** is a normal distribution with a mean of 0 and a standard deviation of 1.

KeyConcept Characteristics of the Standard Normal Distribution

- The total area under the curve is equal to 1 or 100%.
- Almost all of the area is between $z = -3$ and $z = 3$.
- The distribution is symmetric.
- The mean is 0, and the standard deviation is 1.



VIDEO SHARING **The number of videos uploaded daily to a video sharing site is normally distributed with $\mu = 181{,}099$ videos and $\sigma = 35{,}644$ videos. Find each probability. Then use a graphing calculator to sketch the corresponding area under the curve.**

a. $P(180{,}000 < X < 200{,}000)$

The question is asking for the percentage of days when between 180,000 and 200,000 videos are uploaded. First, find the corresponding z-scores for $X = 180{,}000$ and $X = 200{,}000$.

$z = \frac{X - \mu}{\sigma}$ Formula for z-scores

$= \frac{180{,}000 - 181{,}099}{35{,}644}$ $X = 180{,}000$, $\mu = 181{,}099$, and $\sigma = 35{,}644$

≈ -0.03 Simplify.

Use 200,000 to find the other z-score.

$z = \frac{X - \mu}{\sigma}$ Formula for z-scores

$= \frac{200{,}000 - 181{,}099}{35{,}644}$ $X = 200{,}000$, $\mu = 181{,}099$, and $\sigma = 35{,}644$

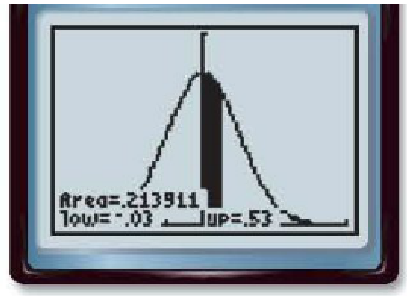
≈ 0.53 Simplify.

The range of z-scores that corresponds to $180{,}000 < X < 200{,}000$ is $-0.03 < z < 0.53$. Find the area under the normal curve within this interval.

Note:

You can use a graphing calculator to display the area that corresponds to any z-score by selecting 2nd [DISTR]. Then, under the **DRAW** menu, select **ShadeNorm(lower z score, upper z score)**. The area between $z = -0.03$ and $z = 0.53$ is about 0.21 as shown in the graph.

Therefore, about 21% of the time, there will be between 180,000 and 200,000 video uploads on a given day.



[−4, 4] scl: 1 by [0, 0.5] scl: 0.125

b. $P(X > 250{,}000)$

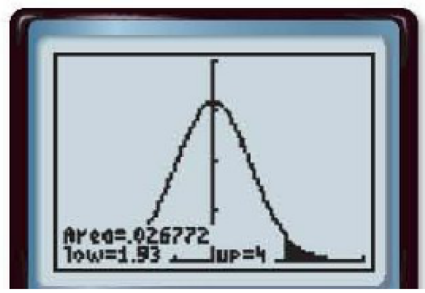
$$z = \frac{X - \mu}{\sigma} \qquad \text{Formula for } z\text{-scores}$$

$$= \frac{250{,}000 - 181{,}099}{35{,}644} \qquad X = 250{,}000,\ \mu = 181{,}099, \text{ and } \sigma = 35{,}644$$

$$\approx 1.93 \qquad \text{Simplify.}$$

Using a graphing calculator, you can find the area between $z = 1.93$ and $z = 4$ to be about 0.027.

Therefore, the probability that more than 250,000 videos will be uploaded on a given day is about 2.7%.



You try:

TIRES The life spans of a certain tread of tire are normally distributed with $\mu = 31{,}066$ miles and $\sigma = 1644$ miles. Find each probability. Then use a graphing calculator to sketch the corresponding area under the curve.

A. $P(30{,}000 < X < 32{,}000)$

B. $P(X > 35{,}000)$

Day 8

Complete the following problems: (use another sheet of paper)

1. The College of Knowledge gives an admission qualifying exam. The results are normally distributed, with a mean of 500 and a standard deviation of 100. The admissions department would like to accept only students who score in the 65th percentile or better. Complete the chart below, and then determine which students would qualify and what score is associated with the 65th percentile. Which students qualify for admission?

Student score	z-score	Percentile
530		
570		
650		
800		
540		

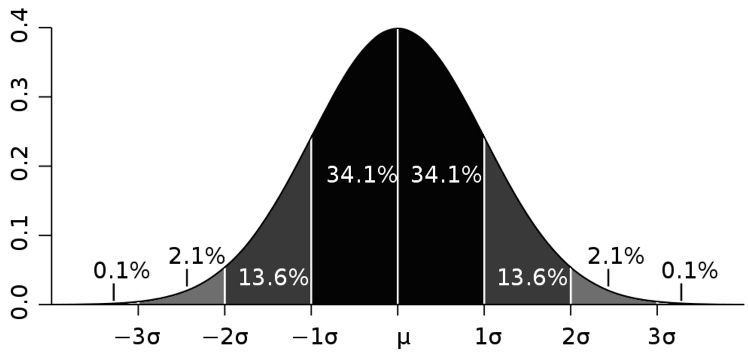
2. The MP3 player, aPod, made by Mango Corp., has an average battery life of 400 hours. Battery life for the aPod is normally distributed, with a standard deviation of 25 hours. The MP3 player, PeaPod, made by Pineapple Inc., has an average battery life of 390 hours. The distribution for its battery life is also normally distributed, with a standard deviation of 30 hours.

- Find the z-scores for each battery with lives of 250, 350, 410, and 450 hours.
- Which battery lasting 410 hours performed better?
- What percent of aPod batteries lasts between 375 and 410 hours?
- What percent of PeaPod batteries lasts more than 370 hours?

Day 9

Normal Distribution Exploration, Part 3

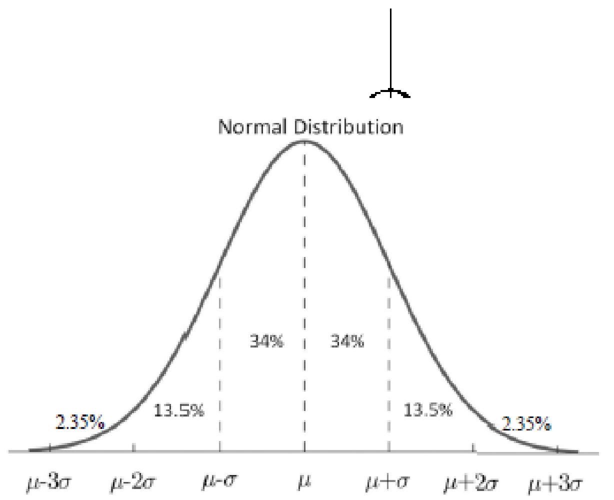
Areas can be found under a normal curve by using the 68-95-99.7 rule if the areas are bounded at places where an exact standard deviation occurs. Areas that are not bounded at specific standard deviation units can be found by using a graphing utility or a z-table.



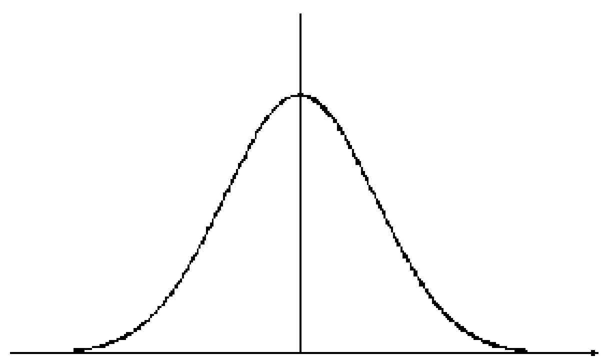
Problem 1

A corn chip factory packs chips in bags with normally distributed weights with a mean of 12.4 ounces and a standard deviation of 0.15 ounces.

1. On the graph at right, label the mean and three standard deviations above and below the mean.



2. Shade the region that indicates the percentage of bags that contains less than 12.64 ounces.
3. Determine the z-score corresponding to 12.64, using the formula z-score = $\frac{x-\mu}{\sigma}$.
4. Use the Standard Normal Probabilities Table to find the area associated with the z-score obtained in 3. Interpret your result.



5. On the graph at right, label and shade the region that represents the likelihood a bag will contain between 12.1 and 12.76 ounces.

6. Calculate the z-scores corresponding to both 12.1 and 12.76, and find the Standard Normal Probabilities for each, using a graphing utility or the Standard Normal Probabilities Table.

7. Explain how you would use those values to determine the probability a bag chosen at random will contain between 12.1 and 12.76 ounces.

Algebra II Learning in Place

Algebra II – Review

Day 10

Factor each polynomial.

$27m^3 + 8$ $\qquad$ $512h^{24} - 343j^{15}$ $\qquad$ $24f^{14} + 192$

$a^6b^{21} + b^3c^{12}d^{30}$ $\qquad$ $3y^{12} - 81$ $\qquad$ $64m^3 - 216$

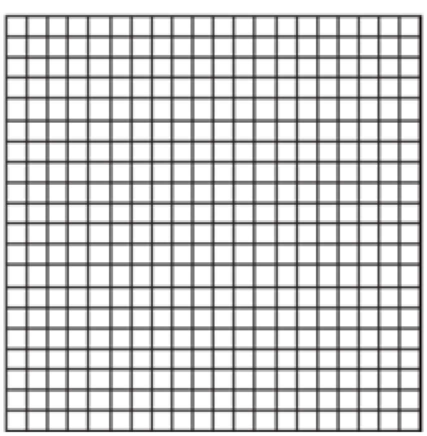
Solve by factoring. Then sketch a graph of the quadratic function based on your solutions.

1.) $2x^2 = 7x - 3$

The factors are ______________________.

The roots/solutions are ________________.

Graph

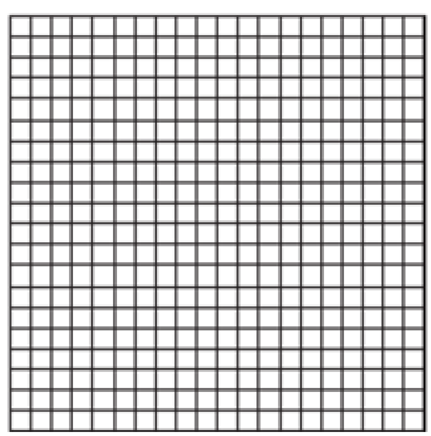


2.) $x^2 + x - 6 = 0$

The factors are ________________________.

The roots/solutions are _________________.

Graph



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Cut the pieces apart and connect them to show the correct matches.

Glue on paper.